Aggregate Output and Pairwise-Stable Production Networks

Allison Oldham Luedtke*†

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Abstract

I study a model of production network formation and use this model to analyze the effect on aggregate output of a firm losing access to an input. I find that, contrary to economic intuition and alternative model results, when a supply line is removed, aggregate output may increase rather than decrease. I solve the model computationally and simulate it to characterize the types of networks that lead to this increase in output. I find that this is more likely when the re-formed production network is relatively interconnected and when the firm that loses access to its input has more alternatives to choose from. This paper identifies circumstances under which it is more likely that aggregate output increases rather than decreases in the face of a supply chain disruption. This can inform the methods used by researchers and help policy makers identify supply chain disruptions that represent opportunities for economic growth.

Keywords: Network Formation, Production Networks, Aggregate Output

JEL Classifications: C67, D85, E23

1 Introduction

When supply chains are disrupted, they must re-form. When this happens, firms face a new set of circumstances and often make different choices than they did before the disruption. This can lead to sharp changes in the total amount produced by all of the firms in the economy. While it may seem intuitive that the disruption of a supply chain should only lead to a decrease in aggregate output, I show in this paper that this may not be the case and identify characteristics of the production environment that make it more likely that output increases. As supply chain disruptions become more common and more globally relevant,[1] it

*Assistant Professor of Economics, St. Olaf College, Northfield, MN 55057. Dr. Luedtke can be reached by email at luedtk2@stolaf.edu.

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[1]When Hurricane Harvey hit the Gulf Coast of Texas in 2017, the oil refineries located there were shut down, forcing American oil manufacturers to switch to refineries elsewhere in the US. [17] In 2018, Toyota factories in the United States were forced to temporarily halt production because an earthquake in Japan had damaged factories there that produce parts needed in the American factories. [8] Early in the coronavirus pandemic in 2020, meat packing plants were closed, preventing restaurants and grocery stores from obtaining meat, especially beef and pork. As a result of this, restaurants were forced to redesign their menus. [16]

is critical to better understand the behavior of endogenous production network formation and its effect on output.

I use a model of production network formation in which firms choose from a set of available input suppliers, forming an equilibrium network to study the effect of a shock to the available suppliers. I investigate the effect on aggregate output when a particular supply line is broken, i.e., when an input supplier is no longer available to a firm. I find that, in contrast to results of models in which all firms do not have the opportunity to make new decisions, aggregate output does not always fall when a firm loses access to an input. The total output produced by the network may, in fact, increase. I show this analytically and then solve the model computationally to identify characteristics of production environments that make this counter intuitive outcome more likely.

In this model of production network formation, firms simultaneously choose input suppliers from amongst the other firms. These input choices form the links of the equilibrium production network. Firms' input decisions affect their costs, profits, and the prices that they charge other firms. As a result, each firm's input choice affects other firms' input choices, as well. The production environment defines a set of possible equilibrium networks. For computational reasons, I restrict the production environments considered in the model to those in which each firm chooses only one supplier. While this is not the case in practice, observed input-output tables suggest that, at least at the sectoral level, the average in-degree is small. [7] Each possible equilibrium network defines a profit maximization problem for each firm. Firms choose their input suppliers based on the profits that result from these maximization problems. Specifically, I find the pairwise-stable equilibrium network that produces the largest aggregate output for a given production environment. A pairwise-stable network is a network in which only mutually beneficial buyer-supplier relationships exist in the equilibrium network. This is a standard definition of an equilibrium network used in the literature. [13, 12, 15, 11] The amount that each firm produces depends on their input choices and the input choices of the other firms, so each possible equilibrium network produces a different level of aggregate output.

I show that when a pairwise-stable equilibrium production network re-forms after a supply line breaks (a link in the network is removed), the output produced by the network may increase. This occurs because, when the supply line is removed, it changes the choice environment for all of the firms in the network. In a pairwise-stable equilibrium production network, the firms do not internalize the overall increases and decreases in output. They make decisions based only on their own profits. Specifically, output increases when the supply line that is removed was the only available deviation preventing a new network from being an equilibrium. When that link is deleted, the resulting equilibrium network may have higher output. This occurs when one buyer-supplier pair is made better off at the expense of lower output in the economy as a whole. When this buyer-supplier relationship is no longer possible, the higher-output network is now stable to all available deviations and therefore an equilibrium. This result is distinct from models in which the network is not allowed to re-form or in which a limited set of firms are allowed to change their input choices when an individual firm loses its input.

To better understand these theoretical results, I present a specific example of the general model and I solve the model computationally. I simulate the model for many randomly generated sets of input options and the results of these simulations suggest network characteristics which are correlated with the event that output increases when an supply line is removed. The results indicate that, while the probability of output increasing is small (about 10%), the probability that output increases is higher when the production network is less connected before the supply line is removed and more connected after the supply line is removed.

These results are in line with existing research that indicates that network structure affects aggregate output (for example, Acemoglu and Azar (2020) [1]), but expands the results in this area to show that production networks do not always affect aggregate output in an intuitive way.

In addition to investigating the role of the connectivity of the network as a whole, I also investigate the role of the centrality of the individual firm that loses its input supplier. The results of the simulation indicate that aggregate output is more likely to increase when the firm that loses its input has more alternative suppliers from which to choose a new input. The increase in output is also more likely when the firm has relatively few customers before the input is removed and relatively many customers after the input is removed. These results taken together indicate that output is more likely to increase after an input is removed when the production economy is less interconnected before the input is removed and more interconnected after the input is removed. The intuition is this: more interconnectedness is a sign of a healthy production economy. When a production network starts out relatively disconnected, it has room for improvement. If it becomes relatively interconnected after the change in inputs, it is more likely that output improves.[2]

This paper contributes to the growing literature on the fragility of production networks and their effect on output. Elliot, Golub, and Leduc (2022) show that in a model of endogenous network formation, aggregate output is sensitive to small aggregate shocks to the economy. [9] I show that aggregate output is also sensitive to small microeconomic shocks to input options in which only one firm in the economy faces a shock. Oberfield (2018) describes a model of endogenous production network formation in which a continuous mass of firms form an equilibrium network that is stable to deviations by any group of firms. [15] This paper describes a special case of the model described in Oberfield (2018) in which the set of firms is finite. The finiteness of the set of firms create the opportunity to solve specific parameterizations of the production model and find the associated equilibrium networks. While Oberfield (2018) uses the model to characterize the emergence of star suppliers and how this affects aggregate output, the focus of this paper is instead on the application of the removal of a supply line. On the other hand, Taschereau-Dumouchel (2022) explores a network with a finite set of firms, but in which the entry and exit decisions of firms drive the network structure, rather than changes to firms' input decisions. [18] Many authors analyze how production network structure relates to output. [3] Like these papers, I analyze how firms' connections affect output, but this paper differs in that it considers how firms behave when an input choice is no longer available and the network must re-form.[4]

Disruptions to supply chains affect all sectors of the economy and can reach all the way around the world. It is important to better understand how these disruptions will affect - among other things - aggregate output. In order to accomplish this, it is critical to take into account the re-formation of the production network and to understand the implications of the formation process. Without doing so may produce not just quantitatively incorrect results, but qualitatively incorrect results, as well. The aggregate output produced by production networks may behave contrary to existing intuition and theories. This paper provides insight into this behavior and characterizes production environments in which it is more likely that aggregate output

[2]While I restrict my focus to pairwise-stable equilibrium networks in this paper, it has been shown that features such as short path distances survive under other definitions of equilibrium. [10]

[3]See for example, Acemoglu et al. (2020), Carvalho and Voigtländer (2014), Acemoglu et al. (2012) and Billand et al. (2019)[1] [6] [2] [5].

[4]Endogenous network formation models have been used to analyze other important aggregate outcomes. Kurosaka (2020) and Allouch and King (2019) study how network structure contributes to the provision of public goods, rather than aggregate output. [14] [3] Battaglini et al. (2021) study how the endogenous formation of social links among legislators determine legislative productivity in the US Congress. [4]

will increase when a supply line breaks. This information can contribute to better policies and firm decisions in the face of increasing supply chain uncertainty. A better understanding of the types of production environments that could lead to increases in output may allow policy makers to identify which supply chain disruptions represent likely decreases in aggregate output and those that represent opportunities for economic growth.

2 Model

Consider a set J containing n firms. These firms are connected to one another by supply relationships. Each firm produces a unique good and chooses an input from the set of goods produced by the other firms. Let $S_j \subseteq J$ be the set of input goods that firm j can use in its production process, called the *input set* for firm j. In equilibrium, there is a link from a firm, i, to a firm, j, if firm j uses firm i's input to make its product.

The links between firms form a production network. For ease of exposition, I restrict the focus of this paper to networks in which each firm chooses exactly one input. Therefore, I consider only networks such that each node has an in-degree of 1.[5]

Definition 2.1. *A feasible network is any network consisting of n nodes and n links, such that the in-degree of each node is* 1. *Label the set of such networks $\mathcal{F}$.*

For a given set of firms, the set of feasible networks is determined by the input sets of all of the firms. A feasible network is constructed by choosing one item from S_j for each firm j. Therefore, the number of feasible networks will be $\prod_{j \in J} |S_j|$. For example, if each firm is able to use the product made by every other firm in production (i.e., $S_j = J \setminus \{j\}$ for each firm j) then the number of feasible networks is $(n-1)^n$.

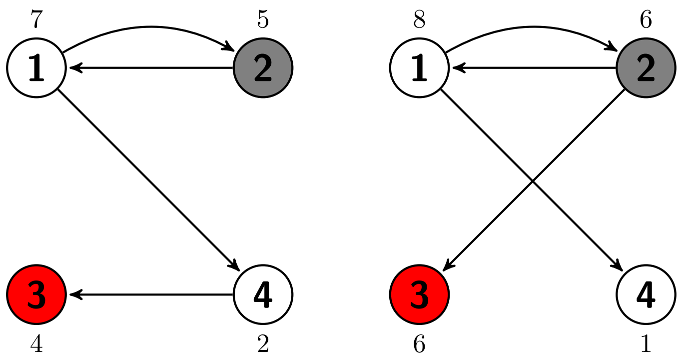
Let there be a function that maps each feasible network to a set of profits, one for each firm. For each feasible network, $N \in \mathcal{F}$, label the set of profits for each firm $\{\pi_j\}_{j \in J}^N$. I make no assumptions about the utility generating process here because none are necessary for the primary results. However, I specify a profit maximization problem for each firm in each network in the Section 4.

Let the total output produced by a network, N, be denoted, Y^N. Let this output be a strictly increasing, continuous function of the vector of each firm's profit in network N, π^N

A *pairwise-stable equilibrium network*, N^*, is a network such that no buyer-supplier pair of firms - between which no link exists in N^* - could both see higher profits by switching to the alternative network $\widetilde{N}$ in which there is a link between those two firms. [12] The following figure depicts a network, N_3, that is not pairwise stable. The profits that each firm earns in each network are written above and below the nodes representing each firm. Firms 2 and 3 both see higher profits in the alternative network, N_4, therefore Firms 2 and 3 have a profitable deviation from N_3 and therefore N_3 is not pairwise stable.

[5] Note that this assumption does not have any implications for the out-degree of a given node.

Figure 1: N_3 is not Pairwise-Stable



(a) N_3 (b) N_4

Figure Note: In this figure, network N_3 is not pairwise-stable.

A given potential network may have multiple pairwise-stable equilibrium networks. Label the set of feasible networks that are pairwise stable, $\mathcal{F}^*$. Each such pairwise-stable network produces a unique level of aggregate output.

Definition 2.2. *The highest-output pairwise-stable equilibrium network (HOPS equilibrium network), is the pairwise-stable network with the largest aggregate output.*

$$N^{HOPS} = argmax_{N \in \mathcal{F}^*}\{Y^N\}$$

Label the output produced by this network Y^{HOPS}*.*

This refinement of pairwise stability defines a unique equilibrium network for any set of firms, their respective input sets, and their respective profits.

3 Network Re-Formation and Output Changes

Let there be a link from firm i to firm j in the HOPS equilibrium network. Suppose that this link is removed from S_j, i.e., firm j can no longer use firm i's product in its production process. A new equilibrium network must form as a result of this change. The removal of this link from the option set necessarily shrinks the set of feasible networks and may change the set of pairwise-stable equilibrium networks and the HOPS equilibrium network. Let $\widehat{\mathcal{F}}$ be the set of feasible networks after this link is removed from the potential network. This is a strict subset of $\mathcal{F}$ because an item has been removed from the input set of firm j, S_j. As a result of this change in the set of feasible networks, the set of pairwise-stable equilibrium networks may change. In fact, it may include feasible networks that were not pairwise stable before the link was removed because the set of deviations that must be considered has changed. This new set of pairwise-stable networks may include a new HOPS equilibrium network with a higher output level than the original HOPS network did.

Condition 3.1. *Let* N^* *be the the HOPS network of some set of feasible networks,* $\mathcal{F}$*. Additionally, let there be a link from firm* i *to firm* j *in* N^**.*

Condition 3.2. *Let there exist another network, $\widehat{N}$, in the set of feasible networks, $\mathcal{F}$, with the following characteristics:*

1. *Firm i is not the supplier for firm j.*
2. *There exists for firms i and j a profitable deviation to another network, $\widetilde{N}$, from $\widehat{N}$, in which i is a supplier for j. That is, $\pi_i^{\widetilde{N}} > \pi_i^{\widehat{N}}$ or $\pi_j^{\widetilde{N}} > \pi_j^{\widehat{N}}$.*
3. *There exist no other profitable deviations from this network to any other networks for any other buyer supplier pairs.*

That is, $\widehat{N}$ would be pairwise-stable if it were not for this one profitable deviation between firms i and j.

Condition 3.3. *Let the aggregate output produced by $\widehat{N}$ is larger than that produced by N^*. That is, $Y^{\widehat{N}} > Y^{N^*}$.*

It must be the case that either firm i or firm j (or both) see a drop in their profits as a result of this change to the set of input options. However, the other firms may see a large enough increase in their profits that the aggregate output measure increases, despite the drop due to firms i and j.

Theorem 3.4. *Under conditions 3.1, 3.2, and 3.3, if firm i is removed from the set of firm j's input options - the link from i to j is removed - the network $\widehat{N}$ will be the new HOPS network with a larger total output than the previous HOPS network, N^*.*

Proof. If firm i is no longer an input option for firm j, then the deviation specified in Condition 3.2 is no longer an option. Because of the first element of Condition 3.2, the network $\widehat{N}$ is in the new set of feasible networks $\widehat{\mathcal{F}}$. As a result of the second element of Condition 3.2, there are now no profitable deviations from $\widehat{N}$ for any buyer-supplier pairs. Therefore, $\widehat{N}$ is pairwise stable. As a result of Condition 3.3, there is now a pairwise-stable network in the new feasible set $\widehat{\mathcal{F}}$ with a larger output than the previous HOPS network. Thus, the aggregate output produced by the new HOPS network is larger than that of the HOPS network before the link was removed. □

In the next section, I provide a specific example of the model described here and simulate the model to provide insight into the network characteristics that make this increase in aggregate output more likely.

4 Example and Simulations

In this section, I describe a specific utility generation process, show that aggregate output may increase under reasonable function forms, and simulate the model to analyze the network characteristics that make this increase in aggregate output more likely to occur. Specifically, I analyze the relationship between this likelihood and the overall connectedness of the production network and three firm characteristics - number of available suppliers, number of firm-customers before the link from firm i to firm j is broken, and number of firm-customers after this link is broken. Simulations indicate that both firm characteristics and overall network characteristics play a role in aggregate output increasing.

4.1 Example

Here I specify a profit maximization problem that generates the firm profits which determine equilibrium networks, as described in Section 2.

Following notation similar to that in Oberfield (2018), let there be a finite set, J, of firms indexed $j = 1, \dots, n$ and each firm produces a single distinct good. [15] All of the firms' output is consumed by either other firms as inputs or by a representative consumer. The *potential network* describes all of the input options available to each firm. In the potential network, there is an edge from node i to node j if firm j can use firm i's product as an input. Each edge, e, from i to j has a weight, $z(e)$, which describes the technological match of that input in firm j's production. Each input available to firm j, that is, each edge pointing to j in the potential network, defines a specific production function that depends on the edge weight, $z(e)$. The amount of firm j's product that it can produce using the input specified by edge e is

$$y_j = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e) x(e)^\alpha l_j^{1-\alpha}$$

where $x(e)$ is the amount of the input that firm j uses and l_j is the amount of labor that firm j uses. The parameter, α, is common across all of the firms and input options.

Let y_j^0 be the amount of y_j that is consumed by the representative consumer. The representative consumer has preferences over the goods produced by the firms in J,

$$U(y_1, \dots, y_{|J|}) = \Big(\sum_{j \in J} (y_j^0)^{\frac{\epsilon-1}{\epsilon}} \Big)^{\frac{\epsilon}{\epsilon-1}}$$

and she supplies L units of labor, inelastically.

Firms set prices for the portion of their output consumed by the representative consumer and the portion consumed by each of their network customers - the other firms which use their good as an input. Label the price of that the consumer pays for each unit of j's product p_j^0. For each network customer of firm j, assume that j charges a per-unit price equal to their marginal cost. That is, j sets $p(e)_{e \in \hat{D}_j}$, for each $\hat{D}_j$ defined by each $N \in \mathcal{N}$ such that $p(e) = MC_j$. As a result of this, the per-unit price that j charges is a function of the marginal cost of the input supplier used by firm j, $p(e) = MC_j = \frac{1}{z(e_j)} MC_{s(e_j)} w^{1-\alpha}$, where w is the price of labor to all firms. Similarly, the price charged by that supplier is a function of the marginal cost of *their* supplier, and so on.

When exactly one edge is pointing to each firm, there are only two possible network shapes that can make up each connected component of the entire network. These are cycles with branches. A cycle is a set of nodes such that the in-degree and out-degree of each node are both exactly one. A branch is a set of nodes such that the in-degree of each node is one but the out-degree is unrestricted. Any connected component must contain exactly one cycle, and any branch in that connected component must have its root on the cycle.[6] Because the price charged by each firm can be written in terms of the supplier's marginal cost, all of these prices can be calculated using only the network structure and $z(e_j)$'s. See the Appendix for a detailed explanation of the price calculations.

With these prices taken as given, the profit maximization problem each firm j solves is

$$\max_{p_j^0, y_j^0, x(e_j), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} p(e) x(e) - p(e_j) x(e_j) - w l_j$$

[6] If it had no cycle then there would exist one node with no supplier, and if there was more than one cycle then there would exist some node with more than one supplier.

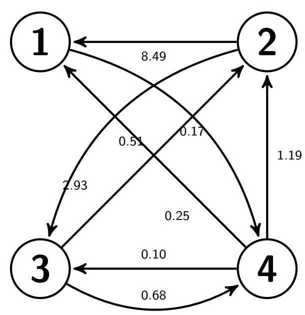
s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e_j) x(e_j)^\alpha l_j^{1-\alpha}.$$

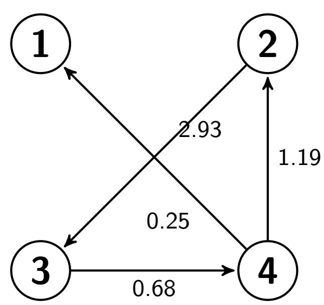
Each $N \in \mathcal{F}$ defines a different profit maximization problem for each firm and the solutions to these maximization problems produce a set of profits for each firm in each feasible network.

Using the parameterization described in the Appendix, the following figure depicts an example in which removing an input option leads to larger aggregate output. Figure 2(a) depicts the set of firms and their available input sets, as well as their pairwise match parameters, as edge weights. The HOPS network for this set of firms is depicted in panel (b). When firm 4's product is no longer an available input option for firm 1, the new HOPS equilibrium network - depicted in panel (c) - produces a higher aggregate output.

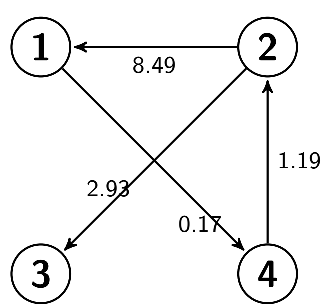
Figure 2: Output increases when the edge from firm 4 to firm 1 is deleted.



(a) Potential Network



(b) Original HOPS Equilibrium Network, Output = 0.1430

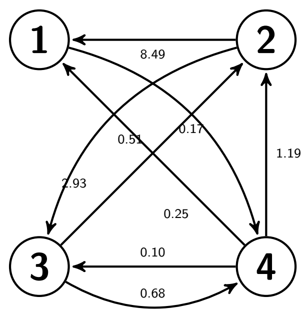


(c) New HOPS Equilibrium Network, Output= 0.1904

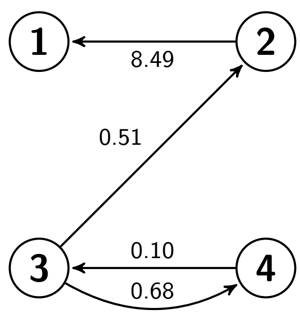
Figure Note: This figure shows an example of output increasing after a link is broken.

This result is distinct from those that arise in a model in which all firms do not have the opportunity to re-optimize. Without allowing for the endogenous re-formation of the production network, we might expect the new network to be the same as the old network with only one change: the firm that lost its input chooses its next lowest marginal cost input. However, such a new network is not necessarily an equilibrium. See Figure 3 for an example. Figure 3 (a) shows the potential network and Figure 3 (b) shows a pairwise-stable equilibrium network. If the link from firm 2 to firm 1 is deleted and the other edges are held fixed, while firm 1 chooses the lowest marginal cost supplier available to it - firm 4 - then the network will be as shown in Figure 3 (c). However, this network is not pairwise stable.

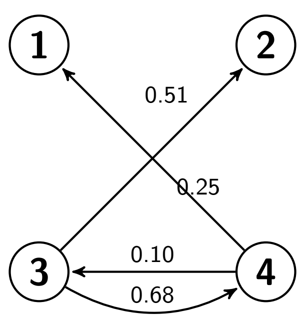
Figure 3: Deleting an edge and holding the other edges fixed is not necessarily an equilibrium



(a) Potential Network



(b) Original Equilibrium Network



(c) Holding other edges fixed

Figure Note: *This figure shows an example of a network that is not allowed to entirely re-form endogenously is not an equilibrium.*

When production networks re-form in the face of new circumstances, it is important to use a model that allows all firms to re-optimize.

4.2 Simulation Results

I simulate this model to understand what network characteristics play a role in aggregate output, and in particular what characteristics play a role in an increase in output when a link is broken. I do this by generating potential production networks and then, for each, finding the highest-output pairwise-stable equilibrium. I create the potential network by drawing a number of possible input suppliers for each firm from a Poisson(3) distribution. The identity of each supplier is drawn uniformly with replacement from the other firms. The productivity parameter, $z(e)$, for each link e is drawn from a Pareto$(0.2, -1.8)$ distribution. See the Appendix for the details of the parameterization of the model used in this simulation. This parameterization is motivated by the Carvalho (2012) survey on Input-Output analysis. [7] This simulation consisted of the creation of $1,000$ potential networks, each of which consisted of five firms. Enough simulation repetitions were conducted to ensure statistical significance of as many of the coefficient estimates as possible; exceptions are noted below. In 554 of these potential networks, a HOPS network is found both before and after the link is broken.

This simulation process builds a random sample of potential production networks with five firms from the universe of potential production networks consisting of five firms. The number of available input suppliers, the identities of the input suppliers, and the pairwise match values (edge weights) are created using a Sobol Set quasi-random number generator.

I investigate the role that network connectivity and firm centrality play in the probability that output increases after a supply line is removed from the production network. To understand the role of network connectivity, I consider the average shortest path distance of the potential network (firms and their input sets), the original HOPS network, and the new HOPS network that results after the link is broken. I also include characteristics of j^*, the buyer that loses access to its supplier. Specifically, I consider the number of suppliers available to j^* in the potential production network (the size of j^*'s input set), the number of firms that buy j^*'s product in the original HOPS network (j^*'s original customers), and the number of firms that buy j^*'s product in the new HOPS network (j^*'s new customers). These six characteristics are the explanatory variables in a binary Logistic regression for which the dependent variable is the probability that output increased after the link is broken. The coefficients on each of these explanatory variables, their standard errors, and their marginal effects are reported in Table 1.

Table 1: Logit Regression Results

Characteristic	Coeff.	SE	ME
Potential Network Average Shortest Path Distance	-0.2643	0.5949	-0.0280
Original Equilibrium Average Shortest Path Distance	0.6338	0.1466	0.0671
New Equilibrium Average Shortest Path Distance	-0.2457	0.1713	-0.0260
Number of Possible Suppliers	0.0686	0.0937	0.0073
Original Number of Customers	-0.2993	0.0873	-0.0317
New Number of Customers	0.2391	0.0839	0.0253

The results regarding the connectivity of the production network indicate the following. First, the more connected the potential network is, the higher the likelihood that output will increase after a link is broken. A one-link increase in the average distance of the potential network is associated with a 2.8 percentage point decrease in the likelihood that output increases. Second, the less connected the original HOPS network and the more connected the new HOPS network, the higher the likelihood that output will increase. A one-link increase in the average distance of the original HOPS network is associated with a 6.71 percentage point increase in the likelihood that output increases, while a one-link increase in the average distance of the new HOPS network is associated with a 2.6 percentage point decrease in the likelihood that output increases.

The firm characteristic results indicate the following. First, the more supplier options available to j^*, the higher the likelihood that output will increase after the link is broken. One more supplier is associated with a 0.73 percentage point increase in the probability that output increases. Second, the fewer firms buying from j^* in the original HOPS network and the more firms buying from j^* in the new HOPS network, the higher the likelihood that output increases. One more customer in the original HOPS network is associated with a 3.17 percentage point decrease in the probability that output will increase, while one more customer in the new HOPS network is associated with a 2.53 percentage point increase in the probability that output increases.

5 Conclusion

I apply a model of endogenous production network formation to investigate the effect of a firm losing an input supplier and find that when this happens the resulting aggregate output can be higher than it was before the firm lost its supplier. I solve the model computation and simulation results indicate the following. First, on average, the more connected a production network is, the smaller the decrease in aggregate output will be when a firm loses an input supplier. Second, on average, the more alternative suppliers that firm has, the smaller the drop in output will be when that firm loses its input supplier. Finally, it is more likely output will increase when (1) the firm that loses its supplier goes from having fewer customers before it lost its supplier to having many customers afterwards and (2) the network as a whole goes from less connected before this input is removed to more connected afterward this input is removed.

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6 Appendix

6.1 Price Derivations

Because the price charged by each firm can be written in terms of the supplier's marginal cost, all of these prices can be calculated using only the network structure and $z(e_j)$'s. The price charged by any firm on a cycle can be traced back through each supplier until it is expressed in terms of itself, thus there is a closed form solution for any price on a cycle. The price charged by any firm on a branch can be traced up to the root node of the cycle, which must be on a cycle, thus any such price can be calculated.

Figure 4: The gray nodes form a cycle; the white nodes form a branch.

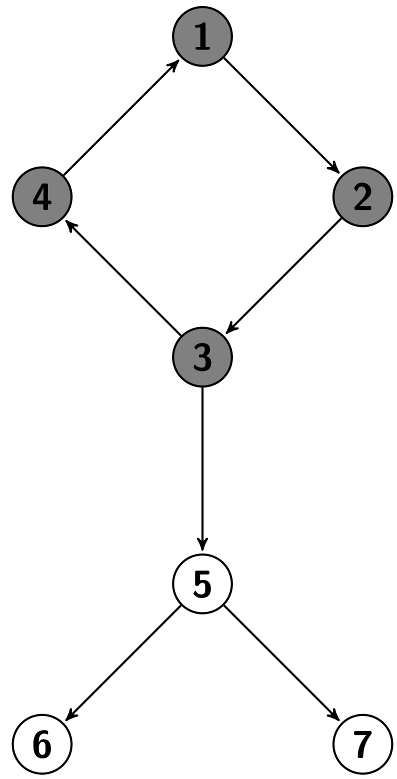

Figure Note: *This figure depicts a network formed of a cycle and branch. The grey nodes form a cycle, the white nodes a branch.*

For a cycle of length c, there are c firms and c edges. Label these firms $1, \ldots, c$. Without loss of generality, we find the price of firm c and label the supplier c uses as 1, the supplier 1 uses as 2 and so on.

Figure 5: Pricing on a Cycle

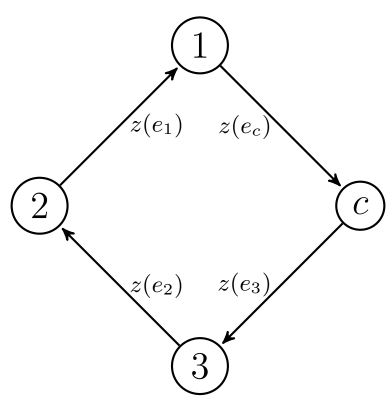

Figure Note: *This figure depicts a production network that is a cycle formed by four firms.*

The price that c pays for its input is $p(e_c) = \frac{1}{z(e_1)} p(e_1)^{\alpha} w^{1-\alpha}$, where $p(e_1)$ is the price firm 1 pays for its input. This price is $p(e_1) = \frac{1}{z(e_2)} p(e_2)^{\alpha} w^{1-\alpha}$, where $p(e_2)$ is the price firm 2 pays for its input. Continuing in this way I can write the price that firm $c-1$ pays as $p(e_{c-1}) = \frac{1}{z(e_c)} p(e_c)^{\alpha} w^{1-\alpha}$. Substituting each price expression into the previous one gives:

$$p(e_c) = p(e_c)^{\alpha^c} \left[\frac{1}{z(e_1)} \frac{1}{z(e_2)^{\alpha}} \cdots \frac{1}{z(e_c)^{\alpha^{c-1}}} \right] w^{(1-\alpha)+\alpha(1-\alpha)+\ldots\alpha^{c-1}(1-\alpha)}.$$

Solving for $p(e_c)$ gives:

$$\begin{aligned} p(e_c) &= \left(\left[\frac{1}{z(e_1)} \frac{1}{z(e_2)^{\alpha}} \cdots \frac{1}{z(e_c)^{\alpha^{c-1}}} \right] w^{(1-\alpha)+\alpha(1-\alpha)+\ldots\alpha^{c-1}(1-\alpha)} \right)^{\frac{1}{1-\alpha^c}} \\ &= w^{\frac{1-\alpha}{1-\alpha^c} \sum_{k=1}^{c} \alpha^{k-1}} \prod_{i=1}^{c} \left(\frac{1}{z(e_{i+1})} \right)^{\frac{\alpha^{i-1}}{1-\alpha^c}} \\ &= w^{\frac{1-\alpha}{1-\alpha^c} \frac{\alpha^c - 1}{\alpha - 1}} \prod_{i=1}^{c} \left(\frac{1}{z(e_{i+1})} \right)^{\frac{\alpha^{i-1}}{1-\alpha^c}} \\ &= w \prod_{i=1}^{c} \left(\frac{1}{z(e_{i+1})} \right)^{\frac{\alpha^{i-1}}{1-\alpha^c}}. \end{aligned}$$

Each connected component of the network has one cycle, and potentially many branches emanating from that cycle. Thus, each branch has a root node on the cycle. Because each firm pays the marginal cost of its supplier, the cost of any branch firm can be traced back and written in terms of the price of this root node. Let firm d be d edges down the branch from the node where $d > 1$.

Figure 6: Pricing on a Branch

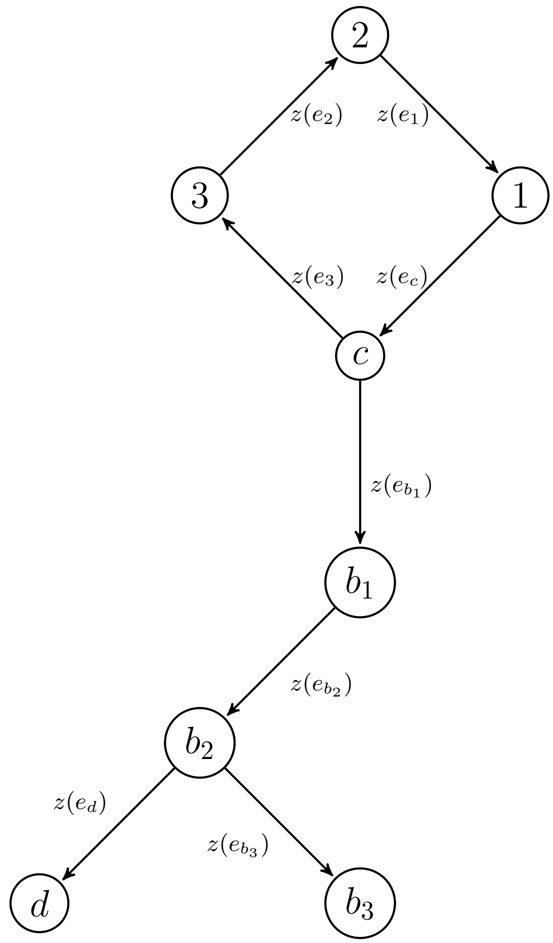

Figure Note: *This figure depicts a production network that is made up of a cycle and a branch.*

The price that d pays for its input is the marginal cost of its supplier. The price the supplier pays is the marginal cost of his supplier and so on up to the root node, whose price was found in the above derivation. Label $MC_r = \frac{1}{z(e_r)} p_r^\alpha w^{1-\alpha}$. Then the price that firm d pays is

$$
\begin{aligned}
p(e_d) &= MC_r^{\alpha^{d-1}} \left[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \right] w^{(1-\alpha) \sum_{k=0}^{d-2} \alpha^k} \\
&= MC_r^{\alpha^{d-1}} \left[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \right] w^{(1-\alpha) \frac{\alpha^d - 1}{\alpha - 1}} \\
&= MC_r^{\alpha^{d-1}} \left[\prod_{i=1}^{d-1} \frac{1}{z(e_{b_i})^{\alpha^{d-i-1}}} \right] w^{1-\alpha^d}.
\end{aligned}
$$

For a firm that is only one edge away, for example firm b_1 in the figure, the price that firm pays is the marginal cost of the root node, MC_r.

With these prices taken as given, the profit maximization problem each firm j solves is

$$\max_{p_j^0, y_j^0, x(e_j), l_j} p_j^0 y_j^0 + \sum_{e \in \hat{D}_j} p(e)x(e) - p(e_j)x(e_j) - wl_j$$

s.t.

$$y_j^0 + \sum_{e \in \hat{D}_j} x(e) \leq \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z(e_j) x(e_j)^\alpha l_j^{1-\alpha}.$$

6.2 Model Parameterization

Table 4: Model Parameterization

Parameter	Value
Number of Firms, $\vert J \vert$	5
Production function capital parameter, α	0.33
Consumer preference parameter, ϵ	0.10
Labor supply, L	1
Number of possible suppliers drawn from	Poisson(3)
Productivity parameters drawn from	Pareto(0.2, −1.8)

6.3 An Algorithm For Computing Pairwise-Stable Equilibrium Networks

1. Initialize the set of pairwise-stable networks as empty.
2. Enumerate the set of feasible networks.
3. For each feasible network:
   (a) Enumerate the pairwise deviations.
   (b) For each deviation:
      i. Check if either of the firms in the pair are made better off.
      ii. If they are, stop.
      iii. If they are not, check the next deviation.
   (c) If any of the deviations are profitable, this feasible network is not pairwise-stable; stop.
   (d) If there are no profitable deviations, this feasible network is pairwise-stable; add it to the set of pairwise-stable networks.
4. Return the set of pairwise-stable networks.