The Transfer Velocity of Money

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Abstract

Monitoring the money supply is an important prerequisite for conducting sound monetary policy, yet monetary indicators are conventionally estimated in aggregate. This paper proposes a new methodology that is able to leverage micro-level transaction data from real-world payment systems. We apply a novel computational technique to measure the durations for which money is held in individual accounts, and compute the transfer velocity of money from its inverse. Our new definition reduces to existing definitions under conventional assumptions. However, inverse estimation remains suitable for payment systems where the total balance fluctuates and spending patterns change in time. Our method is applied to study Sarafu, a small digital community currency in Kenya, where transaction data is available from 25 January 2020 to 15 June 2021. We find that the transfer velocity of Sarafu was higher than it would seem, in aggregate, because not all units of Sarafu remained in active circulation. Moreover, inverse estimation reveals strong heterogineities and enables comparisons across subgroups of spenders. Some units of Sarafu were held for minutes, others for months, and spending patterns differed across communities using Sarafu. The rate of circulation and the effective balance of Sarafu changed substantially over time, as these communities experienced economic disruptions related to the COVID-19 pandemic and seasonal food insecurity. These findings contribute to a growing body of literature documenting the heterogeneous patterns underlying headline macroeconomic indicators and their relevance for policy. Inverse estimation may be especially useful in studying the response of spenders to targeted monetary operations.

Keywords: Velocity of Money, Community Currency, Transaction Networks

JEL Codes: E41, E42, C63, L14

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1 Introduction

The rate at which money changes hands is an important macroeconomic indicator and plays a key role in determining inflation (Benati 2020; Benati 2023). Money tends to move faster between people when the economy is doing well and slower when the economy is doing poorly (Leão 2005). However, the velocity of money has historically been treated as a "black box" that enters into structural models as a measured parameter (Gould et al. 1978); conventional measures are typically constructed as the ratio of two large macroeconomic aggregates. For example, on the Federal Reserve Economic Data (FRED) online database, the Velocity of M2 (FRED ID: M2V) is calculated as the ratio of quarterly nominal gross domestic product to M2, a measure of the aggregate money supply (Federal Reserve Bank of St. Louis 2022). Aggregate measures obscure heterogeneity in the rate at which money changes hands, and the economics literature has long recognized that the velocity of money is likely to be different across sectors (Leontief and Brody 1993; Brody 2000) and between payment systems (Mbiti and Weil 2015). For a single payment system, the *transfer velocity of money* is conventionally calculated as the ratio between the total transaction volume over a period of time and the average total balance of the payment system in that period (Mbiti and Weil 2013).

This paper develops new mathematical and computational techniques that make it possible to measure the transfer velocity of money from micro-level transaction data as recorded by real-world payment systems. We contend that the rate at which money changes hands can be defined more precisely in terms of its inverse, that is, in terms of "holding times." Funds enter an account, are held there for some period, and then are transferred out. In the context of simulated transaction processes where a stationary distribution of holding times can be derived, it is possible to compute the conventional transfer velocity (Wang, Ding, and Zhang 2003; Wang, Ding, and Xi 2005; Kanazawa et al. 2018). Empirical estimation has been developed following this approach (Campajola, D'Errico, and Tessone 2022), but is hampered by the mathematical constraint that the distribution be stationary. We generalize the mathematical definition used in prior work to provide a precise empirical interpretation of the transfer velocity also in cases where spending patterns change. We apply a recent computational technique to measure the durations for which money is held in individual accounts, and compute the transfer velocity of money from its inverse: the average held duration for funds transferred in a period of time.

Inverse estimation of the transfer velocity of money produces precisely defined estimates from digitally recorded transaction data and allows for a fuller empirical analysis of the rate at which money changes hands. We apply this new methodology to a data set of 400,000 individual transactions from a digital community currency in Kenya. *Sarafu* is a community currency managed by Grassroots Economics Foundation, a Kenyan nonprofit organization. The "Sarafu Community Inclusion Currency 2020 - 2021" data set describes every transaction of approximately 40,000 users over a period from January 2020 to June 2021 (Ruddick 2021). This dataset has been thoroughly documented (Mattsson, Criscione, and Ruddick 2022) and used in several prior studies (Ussher et al. 2021; Mattsson, Criscione, and Takes 2023; Ba, Zignani, and Gaito 2023). Since the data describes every transaction from one account to another, we can construct a weighted, directed, temporal network. Then we use an existing technique (Mattsson and Takes 2021) to trace units of currency through the network and extract their holding times in each individual account. Finally, the data contains limited information about the account holders, allowing us to analyze heterogeneity across sub-groups.

Our inverse estimation method gives a value for the transfer velocity of Sarafu that is substantially higher than the conventional estimate, indicating that not all units of the currency remained in active circulation. Specifically, units of Sarafu changed hands at a rate of 0.70 or 0.31 transactions per week, on average, over the full observation period, using inverse or conventional estimation, respectively. Given the observed volume of transfers, the discrepancy implies that 56% of the total Sarafu balance was effectively static. This heterogeneity can be further explored using the empirical distribution of held durations—some units of Sarafu were held for minutes, others for months—and is consistent with prior empirical work. In estimating the velocity of M-pesa in Kenya, Mbiti and Weil 2013 note that "most e-money at any point in time is held by nonfrequent transactors, even though most transfers are done by frequent transactors". Similarly, recent empirical work on cryptocurrencies finds dramatic differences in spending rates across individual accounts (Campajola, D'Errico, and Tessone 2022).

Because the communities using Sarafu experienced substantial economic disruption over the observation period - including the start of the COVID-19 pandemic, spending patterns were far from stationary. This highlights the need for our new methodology, which enables a detailed analysis in this case. Applying inverse estimation by sub-group, we find major differences in the Sarafu system as used in urban Nairobi and in rural Kinango Kwale. The average transfer velocity was especially high for an extended period among those using Sarafu in informal settlements of urban Nairobi, as the COVID-19 pandemic and related mitigation policies brought about acute economic disruption (FEWS NET 2020b). In rural Kinango Kwale, where disruption was less acute and Sarafu was more established, transfer volumes expanded with only a temporary spike in the transfer velocity. The communities using Sarafu in Kinango Kwale also brought older Sarafu back into circulation some months later, likely as a response to seasonal food insecurity. These findings are consistent with existing work on the use of community currencies in periods of economic disruption (Stodder 2009; Zeller 2020; Ussher et al. 2021).

The case of Sarafu provides evidence that spenders in the economy experience macroeconomic phenomena differently. However, the currency management procedures that were in place at the time were system-wide and this may have been limiting[1]. In this vein, we explicitly analyze the effect of a system-wide currency operation conducted in October 2020. Targeted at inactive accounts, this operation removed 27% of Sarafu from circulation in less than two days. This sharp drop in the total balance of the system makes conventional estimates of the transfer velocity of Sarafu especially unreliable around this time. Inverse estimation, on the other hand, allows us to consdier the transfer velocity (and the effective balance) even as this operation took place. Interestingly, we find no discernible response among spenders in Nairobi to removing 27% of the total Sarafu held in accounts in Nairobi. We conclude that the funds held in the targeted accounts, which were largely inactive, had not been accessible to the community actively using Sarafu in Nairobi. This result suggests that there can be consequential heterogeneity also in the response to monetary interventions. Good monetary policy requires monitoring how people experience the economy and, crucially, when those experiences diverge.

Our work contributes to a growing literature that uses new data and empirical techniques to identify and analyze heterogeneity in key macroeconomic measures. With respect to the transfer velocity of money, specifically, Mbiti and Weil 2015 argue that differences between payment systems can have macroeconomic impacts in countries where payment infrastructures are developing rapidly. This has been the case with Mobile Money in Kenya, Uganda, and other countries in East Africa over more than a decade (Mawejje and Lakuma 2017). In recent years, rising interest in digital financial services, cryptocurrencies, and central bank digital currencies (CBDCs) has made heterogeneity in payment systems a relevant topic of study more broadly (Ron and Shamir 2013; Kondor et al. 2014; Badev and Chen 2014; ElBahrawy et al. 2017). There is growing evidence that also other macroeconomic measures, such as inflation and unemployment, and important

[1] Earlier iterations of the Sarafu system favored decentralized currency management procedures, and as do subsequent systems in place since April 2022 and July 2023. The highly centralized system studied here was intended as a stop-gap technology; Grassroots Economics changed priorities when the COVID-19 pandemic arrived in Kenya.

macroeconomic channels such as monetary policy, affect individuals, groups, and locales differently (Argente and Lee 2021; Gornemann, Kuester, and Nakajima 2021; Bartscher et al. 2021). Advancements include the use of micro-level retail transaction records from payment processors to provide more timely and more geographically precise insight into consumer spending Aladangady et al. 2019. In synthesizing much of the literature on heterogeneity in unemployment, inflation, and economic growth, Goodman-Bacon 2021 argues that without understanding the ways that different groups and individuals experience the economy, macroeconomic policy-makers cannot realize the full potential for economic growth. We expand this literature to include a consideration of heterogeneity in the transfer velocity of money.

In the next section, we describe the Sarafu dataset and the information contained therein. In Section 3, we describe our methods, both theoretical and computational. Section 4 describes the results of our empirical investigation. Finally, Section 5 concludes.

2 Data

Sarafu is a small and substantially re-transacted digital community currency in Kenya. The Sarafu system is operated by Grassroots Economics Foundation, a Kenyan nonprofit organization whose efforts are concentrated in specific places and aimed at supporting marginalized, food insecure communities. Payments in Sarafu are made via a mobile phone interface and one unit of Sarafu is roughly equivalent in value to a Kenyan shilling. Grassroots Economics has made a portion of the system's administrative records available for research. The published dataset includes anonymized account information for around 55,000 users and records of all Sarafu transactions conducted from 25 January 2020 to 15 June 2021 (Ruddick 2021).

Mattsson, Criscione, and Ruddick 2022 describe the 2020-21 Sarafu dataset in detail. Transactions totaling around 300 million Sarafu capture various economic and financial activities such as purchases, remittances, and participation in savings and lending groups. Most user activity is recorded as ordinary transfers, identified in the data as STANDARD transactions. We also consider AGENT_OUT transactions to be "transfers" in that they indicate purchases, of sorts, that facilitated donations to savings and lending groups; there were limited instances of this. In general, it was not possible to exchange Sarafu with Kenyan Shillings. This means that Sarafu was a small, almost completely closed system with respect to the national currency.

Mattsson, Criscione, and Ruddick 2022 also describe the currency management practices and administrative procedures that were in place over the course of the observation period. The creation and removal of Sarafu took place via DISBURSEMENT and RECLAMATION transactions, respectively. New units of Sarafu were systematically issued to newly created accounts. Later in the observation period, so-called demurrage charges served to remove a small fraction of existing Sarafu each month. Besides routine currency management, various administrative operations were undertaken at various times. The accounts involved in currency management and administrative operations are identified in the data with a *system* label. We include in this category also the account labeled *vendor*, since it was used to facilitate pandemic aid. Values reported in our figures and tables do not include the contribution of *system* accounts.

The observation period covers the first year of the COVID-19 pandemic and includes several documented pilot projects, interventions, and currency operations (Ussher et al. 2021; Mattsson, Criscione, and Ruddick 2022). Figure 1 (left) shows the weekly volumes of Sarafu transfers and the total balance of the system over time. Transfer volumes expanded dramatically as the communities using Sarafu experienced economic disruption related to the COVID-19 pandemic. It is worth noting that this pattern is in line with the prevailing understanding of community currencies, the use of which is thought to be counter-cyclical Stodder

2009; Stodder and Lietaer 2016; Zeller 2020. The total balance of the Sarafu system also grew substantially in the first half of the observation period as new accounts were created. There are noticeable discontinuities where specific currency operations added or removed an appreciable fraction of the total balance.

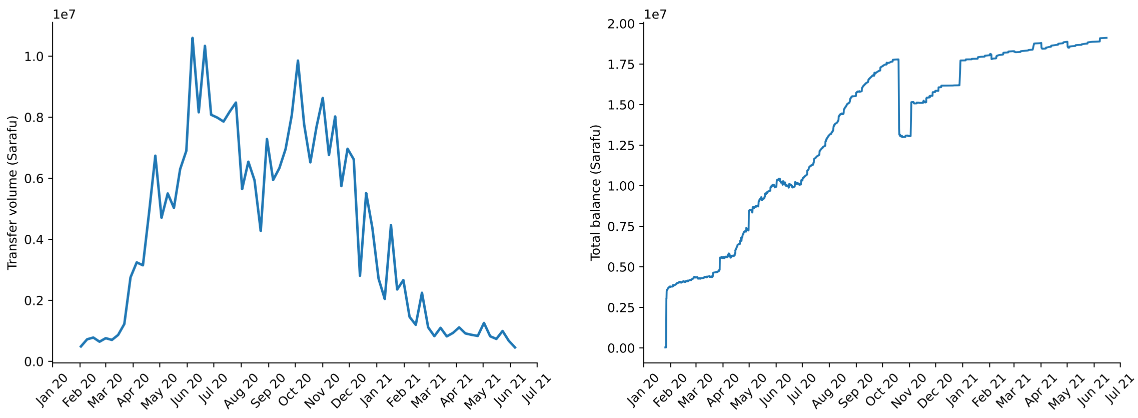

Figure 1: Weekly volume of Sarafu transfers (left) and the total system balance over time (right).

The anonymized account data also contain several contextual attributes that describe the characteristics of account holders. The "area name" and "business type" are user-generated entries generalized into broader categories by staff at Grassroots Economics. They reflect the home location of the user and the product category of the goods or services they provide to the community. Localities are also categorized into *urban*, *periurban*, and *rural* "area types." Mattsson, Criscione, and Ruddick 2022 provide precise descriptions of these data fields and Ussher et al. 2021 provide further detail on their possible values.

Prior work with the 2020-21 Sarafu dataset has established that Sarafu was acting as a community currency in several areas of Kenya, seeing mostly local circulation and considerable involvement from community-based institutions. The structure of circulation within the Sarafu system over this period was highly modular and geographically localized, yet occurring among users providing diverse products (Mattsson, Criscione, and Takes 2023). The explicit involvement of community-based institutions, especially savings and lending groups, is an innovative aspect of the Sarafu system. There are a few hundred *group accounts* noted in the data, and they play an important role (Ba, Zignani, and Gaito 2023). Moreover, digital community currencies are seen as a promising new modality for humanitarian aid. Ussher et al. 2021 use the example of Sarafu over this period to argue that digital community currencies compare favorably to cash assistance because they help establish local economic connections that keep money circulating within a community.

3 Method

In this section we present and reconcile two existing definitions of the transfer velocity of money, clarifying the underlying assumptions. Section 3.1 introduces an expression for the quantity theory of money as it applies to a single, individual payment system. It is noted that the total volume of transfers over a period of time can be decomposed into an independently observable transfer velocity and an "effective balance", which is not necessarily equal to the total balance of the system. This extends tractability to modern digital systems where currency is routinely, and instantaneously, created and removed. In Section 3.2 we recount the steps taken to estimate the transfer velocity of money using the conventional measurement methodology. We present our methodology for inverse estimation in Section 3.3. Section 3.4 explains how this enables us to study heterogeneities along several dimensions. Finally, we detail our implementation in Section 3.5.

3.1 Theoretical Definition

The transfer velocity of money is conventionally defined using an identity similar to that expressing the quantity theory of money. The total transfer volume or the total flow of money, F_T, over a period of time, $T_0 < t < T_1$, is related to the amount of money in circulation, M, and its (average) transfer velocity, V. Equation (1) describes the familiar relation. We include the duration $T = T_1 - T_0$ in our formulation, explicitly, so that both F_T and M are amounts denoted in units of currency.

$$F_T = M \cdot T \cdot V \tag{1}$$

The transfer velocity can also be defined using the concept of a "holding time." Denoted τ, holding times are the durations between when accounts receive and re-transact particular units of money. Wang, Ding, and Zhang 2003, Eqn. 4 define $P(\tau)$ as the probability density for a given unit of money being used in a transaction after an interval of τ. This could be considered an inter-event time distribution, weighted with respect to the units of money. Integrating over this distribution is to consider all units of money at a snapshot in time, allowing an expression for the transfer velocity (Wang, Ding, and Zhang 2003, Eqn. 8). Restated here:

$$V = \int_0^\infty P(\tau) \cdot \frac{1}{\tau} \cdot \mathrm{d}\tau \text{ where } 1 = \int_0^\infty P(\tau)\mathrm{d}\tau \tag{2}$$

The definitions in Equations (1) and (2) rest on similar assumptions. Both implicitly assume that M is fixed, or, at least, that $M(t)$ is well represented by its time-average (Mbiti and Weil 2013). Moreover, to arrive at Equation (2), Wang, Ding, and Zhang 2003 consider a stochastic transaction process in its stationary state, where $P(\tau)$ is independent of t. Here we introduce $P(\tau, t)$ as the non-stationary generalization of $P(\tau)$. While an inter-event time distribution that itself changes in time would be conceptually difficult to observe for any real system, it is useful in derivations. Equation (3) formulates a generalized version of the expression for F_T without the stationary assumption, that is, where both M and $P(\tau)$ are functions also of t, cf. Wang, Ding, and Zhang 2003, Eqn. 7. Equation (4) uses differential form to express the transfer volume generated by the share of money transacted after a duration of exactly τ, cf. Wang, Ding, and Zhang 2003, Eqn. 6.

$$F_T = \int_{T_0}^{T_1} \int_0^\infty M(t)P(\tau, t) \cdot \frac{1}{\tau} \cdot \partial\tau\partial t \tag{3}$$

$$F_T(\tau) = \int_{T_0}^{T_1} M(t)P(\tau, t) \cdot \frac{1}{\tau} \cdot \mathrm{d}t \tag{4}$$

Notably, $F_T(\tau)$ is empirically observable. Equation (4) lets us define the average duration of time that money was held prior to being used in a transaction over the period $T_0 < t < T_1$. This is the average held duration, denoted as $\bar{\tau}_T$ in Equation (5).

$$\bar{\tau}_T = \frac{1}{F_T} \int_0^\infty F_T(\tau) \cdot \tau \cdot \mathrm{d}\tau \tag{5}$$

We can now show that the average held duration is the inverse of the transfer velocity under the conventional assumptions. Incorporating Equation (4) into Equation (5) produces Equation (6). When $M(t)$ and $P(\tau, t)$ are independent of t, the double integral simplifies to $M \cdot T$. Rearranging the simplified expression gives us Equation (7). Note the parallel with Equation (1), with $\bar{\tau}_T^{-1}$ taking the place of V.

$$\bar{\tau}_T = \frac{1}{F_T} \int_0^\infty \int_{T_0}^{T_1} M(t) P(\tau, t) \partial t \partial \tau \tag{6}$$

$$F_T = M \cdot T \cdot \bar{\tau}_T^{-1} \tag{7}$$

Our new measure for the transfer velocity of money is defined in Equation (8) as the inverse of the average held duration, $\bar{\tau}_T^{-1}$. It is denoted by V_T because we expect neither the balance to be fixed nor the process to be stationary. In practice, our measure V_T is likely to diverge from the conventional measure V.

$$V_T = \bar{\tau}_T^{-1} \tag{8}$$

3.2 Conventional Estimation

Conventional estimation of the transfer velocity V uses Equation (1). In practice, F_T is a directly measurable quantity while M is estimated. Only rarely does the total balance of a system remain unchanged over the period $T_0 < t < T_1$ and it is common to use the time-averaged total balance (Mbiti and Weil 2013). Equation (9) formulates an expression for V using the time-average of $M(t)$. Again, $T = T_1 - T_0$.

$$V = F_T / (M_{\text{avg}} \cdot T) \text{ where } M_{\text{avg}} = \int_{T_0}^{T_1} M(t) \mathrm{d}t / T \tag{9}$$

Values of F_T and $M(t)$ can be measured from large-scale micro-level transaction data. The total flow is the combined amount over transfer transactions that occur in the period $T_0 < t < T_1$. The total balance at time t is the total amount of money held across user accounts in that moment. This may be estimated by subtracting the balance of provider facing accounts from the total system balance (Mbiti and Weil 2013).

3.3 Inverse Estimation

Inverse estimation of the transfer velocity V_T can be done using Equation (5). The distribution $F_T(\tau)$ over the period $T_0 < t < T_1$ can be obtained from large-scale micro-level transaction data, as detailed below, and so the average held duration, $\bar{\tau}_T$, can be computed empirically.

Moreover, we can define the *effective balance* of the system over the period $T_0 < t < T_1$ as the fixed balance that would satisfy the inverse relationship between the average held duration and the transfer velocity of money. The effective balance M_T is defined in Equation (10), drawing from Equation (6).

$$M_T = \frac{1}{T} \int_0^\infty \int_{T_0}^{T_1} M(t) P(\tau, t) \partial t \partial \tau = \frac{1}{T} \cdot F_T \cdot \bar{\tau}_T \tag{10}$$

3.3.1 Empirical Holding Times

This section introduces a computational method for finding $F_T(\tau)$ and $\bar{\tau}_T$. So-called "trajectory extraction" can be used to obtain empirical holding times from digital transaction records. Funds are traced through individual accounts, from transaction to transaction, and this gives the durations τ for which money was held. By considering the durations ending with transfer transactions that occur in the period $T_0 < t < T_1$, Equation (5) can be evaluated as a weighted average.

Trajectory extraction performs a data transformation based in the theory of walk processes on networks (Mattsson and Takes 2021). Transactions out of an account are allocated funds from prior transactions into that account, tracing the flow of funds through sequences of accounts. We consider all directly subsequent pairs of transactions, from an incoming to an outgoing transaction; each pair corresponds to an empirical holding time. In running trajectory extraction, we select the "well-mixed" allocation heuristic so that an amount is assigned to every possible pair. The allocated amounts thereby reflect proportional assignment of funds from prior incoming transactions to outgoing transactions. The timestamp of the outgoing transaction is when the observation takes place and so we favor the past tense: these are "held durations."

Held durations that end in a transaction observed within the period $T_0 < t < T_1$ are collected into an empirical distribution; this is $F_T(\tau)$. The observations are weighted; F_T is the total weight. We estimate $\bar{\tau}_T$ as a weighted average.

3.4 Heterogeneity Along Observable Dimensions

Considering different subsets of holding times makes it possible to study heterogeneities underlying the transfer velocity of money. There is a particular point in time at which a held duration is observed, and a particular account where this money had been held. Because of this specificity, features known or derived at the level of individual accounts can be used to characterize underlying heterogeneity. In the Sarafu data set, accounts are labeled with particular user characteristics as noted in Section 2. Known or derived features of the transaction pair that defines the held duration can also be used. For many analyses we consider the subset of held durations that ended in transfer transactions.

Held durations are grouped based on features of the accounts where the funds had been held, or of the transaction pairs used to define them. An example a resulting sub-group would be all held durations that occurred at accounts with a reported location in "Nairobi". Or, all held durations at accounts with a reported location in "Nairobi" that were observed in March 2020. For each sub-group, it is possible to create an empirical distribution of the held durations and an inverse estimate of the transfer velocity. These can then be compared.

3.5 Implementation

We compute the transfer velocity of money for the Sarafu currency over the period from 25 January 2020 through 15 June 2021. The empirical distributions are produced using the held durations observed over the entire observation period. Weekly and monthly estimates are produced for the weeks and months that fall fully within this period. The total balance of the Sarafu system is computed at every hour as captured by the DISBURSEMENT and RECLAMATION transactions (see Section 2). The balance of *system* accounts is subtracted from the total.

Trajectory extraction is done using `follow-the-money`, an open-source piece of software available at https://github.com/carolinamattsson/follow-the-money (Mattsson 2020). The system specifications are noted in a configuration file included within the supplementary material. Briefly, transaction volumes correspond to STANDARD and AGENT_OUT transactions (collectively referred to as *transfers* throughout this work). The system boundary is precisely defined by the DISBURSEMENT and RECLAMATION transactions. Holding times that took place at *system* accounts are then filtered out.

The parameters used for holding time extraction are noted in a script file included with the supplementary material. Tracing digital funds requires selecting an allocation heuristic, as discussed in Section 3.3.1 and in Mattsson and Takes 2021. We use the `--well-mixed` heuristic together with the `--pairwise` option to consider all possible pairs of sequential transactions. Fragments of received transactions with a size below 0.1 Sarafu are not tracked indefinitely; using a `--size limit` of 0.01 Sarafu or 1.00 Sarafu gives the same estimates at the precision with which we report our main results. Finally, a very small amount of Sarafu is mis-recorded in the data (Mattsson, Criscione, and Ruddick 2022). We employ the functionality provided in `follow-the-money` to infer the existence of missing funds, and find this to be an insignificant source of noise.

Analysis is performed in `pandas` (Reback et al. 2020), and figures are produced using `seaborn` (Waskom 2021) and `matplotlib` (Caswell et al. 2019). Distributions are smoothed using kernel density estimation on a logarithmic scale, utilizing the `seaborn` default method for selecting the smoothing bandwidth.

4 Results

In the conventional sense, Sarafu changed hands at a rate of 0.31 transactions per week, on average, over the full observation period. However, as we will show in the remainder of this section, this value obscures underlying heterogeneity along several dimensions. Section 4.1 presents the inverse estimate of the transfer velocity and the effective balance of Sarafu alongside the highly heterogeneous empirical distribution used to produce these measures. Section 4.2 documents average differences in the measures across communities using Sarafu in different areas of Kenya. Section 4.3 considers inverse estimates of the measures over time, revealing temporal heterogeneity as communities using Sarafu experienced acute economic disruption and seasonal food insecurity. In Section 4.4 we observe the response to a targeted currency operation that removed 26.7% of Sarafu from circulation.

4.1 Distributional heterogeneity

The durations for which Sarafu was held prior to the transactions made in this period vary over several orders of magnitude, revealing strong *distributional heterogeneity*. Figure 2 shows the probability density of observing particular held durations prior to a recorded transaction. Sarafu is often spent the same day it is received, sometimes in a matter of minutes. It is also common for funds to be held overnight and spent later in the same week, or in the following week. At times, Sarafu is held for longer periods before being spent. However, many of the longest held durations were observed prior to a RECLAMATION transaction used to remove Sarafu from circulation. This means that the long-duration tail of the total distribution reflects primarily currency operations, not transfers initiated by spenders.

Our estimate of the transfer velocity of Sarafu is computed from the observed holding times that ended with transactions contributing to the transfer volume. That is, the empirical holding times prior to STANDARD and AGENT_OUT transactions excepting those made by *system* accounts (see Section 2). Transfers were made with funds that had been held for 1.4 weeks, on average, corresponding to a transfer velocity of 0.70 transactions per week. The inverse estimate is higher than the conventional estimate (0.31 transactions per week) in that there are few long-duration holding times ahead of transfer transactions, lowering the average observed held duration. This discrepancy lets us conclude that a substantial portion of the Sarafu balance must have been effectively static. To produce the observed volume of Sarafu transfers, effectively 44% of the total Sarafu balance was circulating at 0.70 transactions per week.

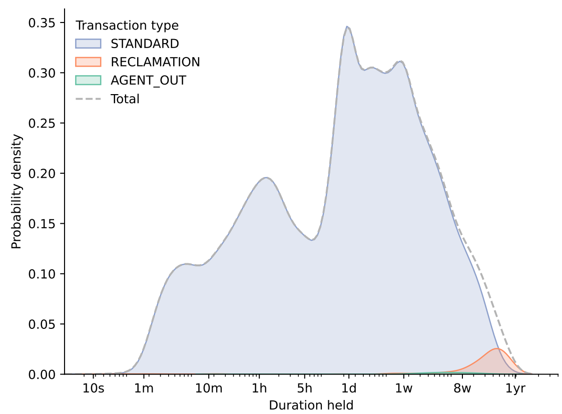

Figure 2: Distribution of the durations for which Sarafu were held prior to transactions observed between 25 January 2020 and 15 June 2021, normalized by the total transaction volume. Contributing to the total distribution (dashed line) are transactions of different types.

4.2 Geographic heterogeneity

Sarafu is a community currency, and circulation was geographically localized over the observation period (Ussher et al. 2021; Mattsson, Criscione, and Takes 2023; Ba, Zignani, and Gaito 2023). Here we document *geographic heterogeneity* in the transfer velocity of Sarafu across the communities using Sarafu in different areas of Kenya. Table 1 lists the inverse estimates of the transfer velocity and other measures computed over the full observation period, separately for the communities using Sarafu in Nairobi, Mombasa, Kilifi, Kinango Kwale, and other rural areas.

Total transfer volumes were especially large in Nairobi and in Kinango Kwale. Nairobi is the capital city of Kenya, and the use of Sarafu was concentrated in informal settlements (especially Mukuru). Kinango is an administrative division in rural Kwale county where Grassroots Economics has had a presence for many years. From Table 1, we see that the community (or communities) in Nairobi sustained a remarkably high transfer velocity of Sarafu. This means that accounts registered in Nairobi tended to hold their Sarafu for shorter periods—spend faster—than those in other areas. In Nairobi, Sarafu changed hands on average three times every two weeks. However, only about a third of the total Sarafu issued and available to the community in Nairobi was consistently in use. Kinango Kwale sustained a higher effective balance, with on average more than half of the total Sarafu balance in use. These funds were changing hands once every three weeks, on average.

Area	Transfer volume (Sarafu)	Period (wks)	Transfer velocity (per wk, avg.)	Effective balance (Sarafu, avg.)	Total balance (Sarafu, avg.)
Nairobi	200.05m	72.35	1.41	1.96m	5.36m
Kinango Kwale	98.49m	72.35	0.36	3.83m	7.35m
Other Rural	2.04m	72.35	0.42	0.07m	0.31m
Mombasa	0.92m	72.35	0.27	0.05m	0.32m
Kilifi	0.96m	72.35	0.36	0.04m	0.20m
Total	302.47m	72.35	0.70	5.94m	13.55m

Table 1: Measures of the circulation of Sarafu in different areas of Kenya. Inverse estimation is used to compute the transfer velocity and the effective balance.

These are, of course, averages over time and over heterogeneous distributions. From Figure 3 (left), we see substantial differences in the temporal pattern of weekly transfer volumes over the observation period for the communities using Sarafu in different areas of Kenya. From Figure 3 (right) we see that much of the transfer volume in Nairobi arose from funds spent within a day, or a week. In Kinango Kwale, much of the transfer volume arose from funds spent within a week, or a month. In the following section we will consider the decomposition of transfer volumes into transfer velocity and effective balance, over time, for the communities using Sarafu in Nairobi and in Kinango Kwale.

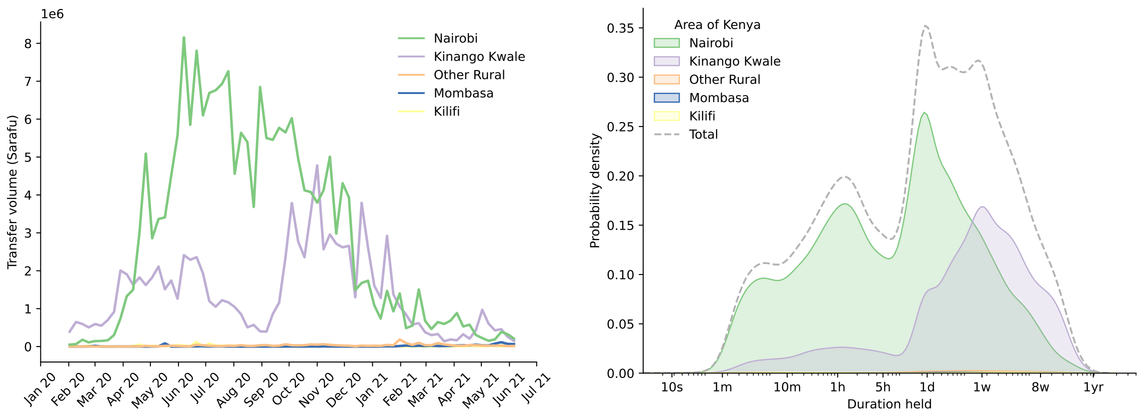

Figure 3: Weekly volumes of Sarafu transfers (left) and distribution of held durations prior to transfers (right) for users registered in different areas of Kenya. The total distribution (dashed line) and constituent distributions are normalized by the total transfer volume.

4.3 Temporal heterogeneity

Changing circumstances can be expected to introduce *temporal heterogeneity* into spending patterns. For instance, beginning in March 2020, widespread behavioral change and national mitigation policies contra the COVID-19 pandemic strongly affected transport, mobility, and business operations in Kenya. This resulted in substantial economic disruption, particularly in poor urban areas (FEWS NET 2020b). As noted in Section 2, the Sarafu system expanded in the following months. Recall that the total system balance grew as new Sarafu was issued to newly registered accounts. In this section, we first consider the experience of users in informal settlements of Nairobi, where Sarafu was being introduced (Mattsson, Criscione, and Ruddick 2022). Then, we consider the experience of a rural community in Kinango Kwale where the Sarafu system had been in use for several years prior to January 2020 (Marion 2018).

4.3.1 Urban Nairobi

The COVID-19 pandemic impacted urban areas of Kenya most directly, prompting widespread behavioral change and a series of official mitigation policies: remote-work directives and nationwide school closures were announced on 15 March 2020; targeted restrictions took effect over the following week and a stringent nationwide curfew was imposed on 27 March; limitations on free movement into and out of Kenya's major cities began on 6 April and remained in effect until 7 July; the nationwide curfew was progressively relaxed on 7 June and on 29 September; bars were allowed to re-open on 29 September; schools were reopened for three grade-levels on 12 October 2020, and fully reopened on 4 January 2021 (Data from www.health.go.ke/press-releases). In acting early, Kenya averted a potentially devastating initial wave of infection. However, the economic impact was substantial and the Kenyan economy shrank in 2020, as measured by Gross Domestic Product (GDP).

The urban poor experienced especially severe economic disruption. Reduced movement and limitations on business operations led to widespread losses in employment and income-generating opportunities for poor urban households. At the same time, mobility restrictions and delays in cross-border trade raised prices for consumer goods including staple foods (FEWS NET 2020b). In phone surveys conducted on 14 April 2020 in informal settlements of Nairobi, 81% of respondents reported complete or partial loss of income (36% and 45%, respectively). Also 87% of respondents reported increased household expenditures, especially on food (*Kenya* 2020; Pinchoff et al. 2021). By June, informal settlements of Nairobi were noted as an *Area of Concern* in the Food Security Outlook for Kenya published by the Famine Early Warning Systems Network (FEWS NET 2020b). Food insecurity persisted through August for many poor urban households and lingered into December for some households, even as the wider Kenyan economy had begun to recover (FEWS NET 2020c; FEWS NET 2020a).

In one informal settlement of Nairobi, a village in the Mukuru slum, the initial disruption coincided with a targeted introduction of the Sarafu community currency (Mattsson, Criscione, and Ruddick 2022). Promotion, education, and training programs managed by the Kenyan Red Cross began in April 2020. Sarafu became part of an improvised humanitarian response effort, apparently meeting an acute need in this community. Use of the Sarafu system expanded dramatically. Figure 4 (left) shows the weekly transfer volume by accounts registered in Nairobi, with periods of more severe restrictions noted in darker shades of grey. Transfer volumes rose by several orders of magnitude in the weeks following the introduction of Sarafu in April and reached a peak in July as restrictions were eased. The use of Sarafu in Nairobi decreased towards the end of 2020, presumably as economic conditions normalized.

Inverse estimation reveals that transfer volumes in Nairobi arose out of intense use of a small subset of the total Sarafu issued and available to this community. Figure 4 (right) shows the weekly and monthly measures of transfer velocity and effective balance for Nairobi. The transfer velocity rose from below one to above three transactions per week in April 2020 and remained exceptionally high into July 2020. Indeed, we can see from Figure 4 (left) that most of the transfer volume in April 2020 and over the subsequent months is attributable to funds held for less than a week before being re-transacted. Though the effective balance also grew substantially over this period, it does not appear to have have kept pace with demand. We consider this an indication that the currency management procedures active over this period in Nairobi were not generous enough, or were not targeted in such a way that the newly issued Sarafu reached those most keen on spending Sarafu.

The frenzied circulation of Sarafu in Nairobi began to slow around the time the most restrictive COVID-19 pandemic mitigation policies were lifted, in July 2020. However, the effective balance continued to grow and transfer volumes remained elevated through November 2020. We consider this an indication that the use of Sarafu broadened over this period. The effective balance of Sarafu circulating in Nairobi began to decline in January 2021, as schools opened fully and the economic recovery appears to have reached also the poorest urban households. Transfer volumes subsided and the transfer velocity, for the smaller share of Sarafu still in use, settled at around one transfer every three to four weeks.

4.3.2 Rural Kinango Kwale

Kinango Kwale is a poor rural area where many experience seasonal food insecurity. Marginal agricultural enterprises provide seasonal work for laborers and own agricultural production provides seasonal support

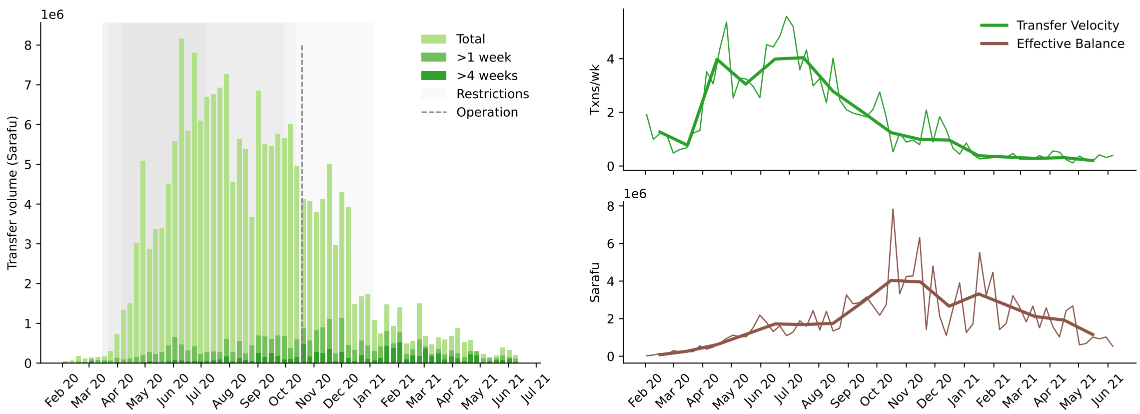

Figure 4: The volume of transfers made each week by users registered in Nairobi (left) and inverse estimates of the monetary indicators, weekly and monthly (right). The share of transfer volumes plotted in darker green are attributable to funds held for at least one or four weeks immediately prior to being transferred. Shaded in grey are periods with more restrictive COVID-19 pandemic mitigation policies. The grey vertical line gives the timing of a currency operation that removed a substantial share of Sarafu from the system.

for subsistence, both subject to climate variability (FEWS NET 2020b; MoALF 2016). It is common for residents to leave for nearby urban areas in search of income-generating opportunities (Marion 2018).

The impact of the COVID-19 pandemic and the nationwide mitigation policies was less direct in Kinango Kwale. Rural livelihoods were indirectly affected via disruptions in the livelihoods of migrant workers in urban areas and changes in the prices of staple foods. Staff at Grassroots Economics liken the economic shock affecting Kinango Kwale during the COVID-19 pandemic to that seen during holiday periods: an influx of migrant workers returning home. In meeting the needs of additional people at higher prices, communities with existing access to the Sarafu system had the option to use the community currency and, as a result, the transfer velocity and the effective balance of Sarafu in Kinango Kwale show considerable temporal heterogeneity.

Figure 5 (left) shows the volume of transfers made each week by users registered in Kinango Kwale and Figure 5 (right) presents inverse estimates of the transfer velocity and effective balance, weekly and monthly. Sarafu saw a sharp increase in use in Kinango Kwale beginning in late March 2020—weekly transfer volumes tripled between February and April. This coincides with a temporary spike in the transfer velocity, suggesting that already-active users stepped up their use of Sarafu as nationwide restrictions entered into effect. High transfer volumes were then sustained for several months in that the effective balance increased steeply. Perhaps because of existing familiarity with Sarafu in the community, large amounts of Sarafu issued to new users reached those keen on using in.

Limitations on free movement were lifted for Kwale county on 7 June, though they remained in effect in Kenya's major cities until 7 July. These developments and, especially, a good harvest in July and August lessened the economic stressors affecting marginal agricultural areas of eastern Kenya (FEWS NET 2020c). Use of Sarafu in Kinango Kwale reached a lull in September, with average transfer velocity and the effective balance falling in tandem. This is a similar pattern as that which occurred in Nairobi when economic conditions normalized, but occurring some months earlier.

Intriguingly, Kinango Kwale also saw a second period of increased use from late September 2020 into early 2021. This swell in transfer volume is of a decidedly different character: we initially see a spike not in the

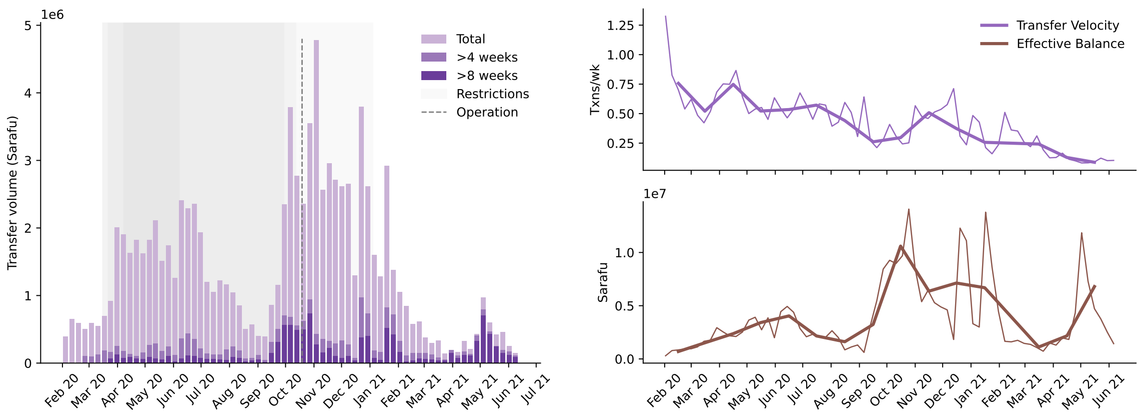

Figure 5: The volume of transfers made each week by users registered in Kinango Kwale (left) and inverse estimates of the monetary indicators, weekly and monthly (right). Transfer volumes plotted in darker shades are attributable to funds held for more than four or eight weeks immediately prior to being transferred. Shaded in grey are periods with more restrictive COVID-19 pandemic mitigation policies. The grey vertical line gives the timing of a currency operation that removed a substantial share of Sarafu from the system.

transfer velocity but in the effective balance, meaning that rising transfer volumes initially reflected renewed use by less active accounts. Indeed, we see from Figure 5 (left) that the volume and share of transfers made with older funds was especially high in late September and October 2020. In the month of October 2020, the effective balance in Kinango Kwale (10.6 million Sarafu) exceeded the average total balance in Kinango Kwale (8.4 million Sarafu). Higher transfer volumes could be sustained over the following months in that the transfer velocity also rose. The effective balance remained high, with monthly averages above 75% of the actual total balance, through the end of the year.

We consider it most likely that this resurgence in the use of Sarafu reflects local seasonality in livelihoods and heightened food insecurity (FEWS NET 2020c; FEWS NET 2020a). This is consistent with the prevailing understanding that community currencies are counter-cyclical, seeing more use in difficult times (Stodder 2009; Zeller 2020). There may be other plausible explanations, and we discuss a complicating factor specific to the Sarafu system in the following section. Absent a comprehensive economic history of this specific area, we conclude only that the experience of the communities using Sarafu in Kinango Kwale in 2020/21 is a superb example of temporal heterogeneity as it applies to payment systems and is reflected in inverse estimates of monetary indicators.

4.4 Response heterogeneity

Here we consider the *response* of the Sarafu system to a specific currency operation. On 19 October 2020, Grassroots Economics initiated a large set of RECLAMATION transactions that removed a substantial amount of Sarafu from the system (Mattsson, Criscione, and Ruddick 2022). This operation accounts for 66% of all Sarafu removed via RECLAMATION transactions over the entire observation period, and these were predominantly long-held funds (see Figure 2 in Section 4.1). The removed Sarafu had been sitting for, on average, 25 weeks in the same account prior to being dissolved. Of the dissolved funds, 89.8% had never entered circulation and only 10.2% had been transferred at least once since coming into existence.

This operation dissolved 26.7% of the total Sarafu balance over two days, visible as a near-instantaneous drop in Figure 1 (right) from Section 2. This would directly affect conventional estimates of the average transfer velocity of Sarafu, because conventional estimates are computed using the time-average of the total issued balance (Equation (9)). Inverse estimation, on the other hand, is not necessarily affected by a change in the total issued balance. Holding times are observed only when users actively make transactions, and, for example, the removal of funds held in inactive accounts might have no impact whatsoever on observed activity in the system.

The October 2020 currency operation was targeted at inactive accounts and, to a lesser extent, accounts with large balances. The direct effect on the communities using Sarafu in Nairobi was of considerable size. A total of 7.6 million Sarafu (27% of Sarafu) was removed in an operation affecting $11,000$ accounts (73% of accounts) registered in Nairobi. The direct effect of the October 2020 currency operation was of comparable magnitude in Kinango Kwale. Moreover, the intention to close out inactive accounts was announced by Grassroots Economics ahead of the operation itself (Mattsson, Criscione, and Ruddick 2022). Our new mathematical and computational method lets us study the response of the communities using Sarafu in these areas.

Inverse estimation reveals the absence of a strong response by spenders in Nairobi to the announcement and implementation of the October 2020 currency operation. Neither the transfer velocity nor the effective balance deviated discernibly from their trends in October 2020 (Figure 4, right). The transfer velocity was falling and the effective balance was rising, both gradually. The transfer volume remained large in Nairobi through September and October 2020, whereas the share of transfers made with long-held funds was and remained small (Figure 4, left). We conclude that the currency operation exposes relevant heterogeneity in the response (or lack thereof) of accounts in Nairobi to the removal of funds.

The use of older funds did increase in Kinango Kwale in late September and early October 2020, resulting in an exceptionally high effective balance in October 2020 (Figure 5). This would be consistent with a robust response to the announcement and implementation of the currency operation. However, the timing of the announcement is not precisely known and we consider seasonal food insecurity to be a more plausible explanation. However, we cannot dismiss the possibility that the rise in the effective balance around this time might be partially attributable to the response by spenders in Kinango Kwale to the October 2020 currency operation.

5 Conclusion

Inverse estimation applies new mathematical and computational methods to measure the transfer velocity of money from micro-level transaction data. We generalize the mathematical definition of the transfer velocity to provide a precise interpretation also for payment systems where spending patterns change over time. Our estimates are computed from the distribution of the durations that money is held prior to being transacted, which can be observed nearly perfectly in digitally recorded transaction data. This method has allowed us to study Sarafu, a community currency system that saw substantial use in several areas of Kenya during the COVID-19 pandemic. We find that much of the transfer volume in Sarafu was driven by quick re-transactions, while a substantial portion of the balance was effectively static. Moreover, there were clear differences in spending patterns between communities in different areas of Kenya and over time. Those using Sarafu in informal settlements of urban Nairobi drove a dramatic increase in the transfer velocity, and in transfer volumes, early in the COVID-19 pandemic. Later on, in rural Kinango Kwale, the re-integration of older Sarafu into active circulation precipitated a second swell of transfer volume. These findings reveal that system-wide monetary indicators comprise distributional, temporal, and geographic patterns of heterogeneity.

This result could suggest that employing system-wide currency management procedures was unduly limiting for Sarafu over the study period. Indeed, Grassroots Economics has since moved to a more localized model of currency management. We also find evidence of heterogeneity in the response to a system-wide currency operation; this is possible because our indicators are not mechanistically affected by the dissolution of money. Inverse estimation can be used in the future to conduct similar analyses on other digital payment systems, such as those operated by mobile payment providers, commercial banks, and central banks. This can lead to improved monitoring of the money supply to better inform macroeconomic policy.

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